

Closing Tue: 10.1/13.1, 10.2/13.2  
Closing Thu: 10.3  
Closing Next Tue: 13.3(part 1)

Midterm 1 is Tuesday, Feb. 2 it covers  
12.1-12.6, 10.1-10.3, 13.1-13.2 and 13.3(part 1)

## 10.2/13.2 Calculus on Parametric Curves (Continued)

Recall:  
For 2D

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(f'(x))}{dx/dt}$$

For 3D

$$\vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

= tangent (velocity) vector

$$|\vec{r}'(t)| = \text{speed}$$

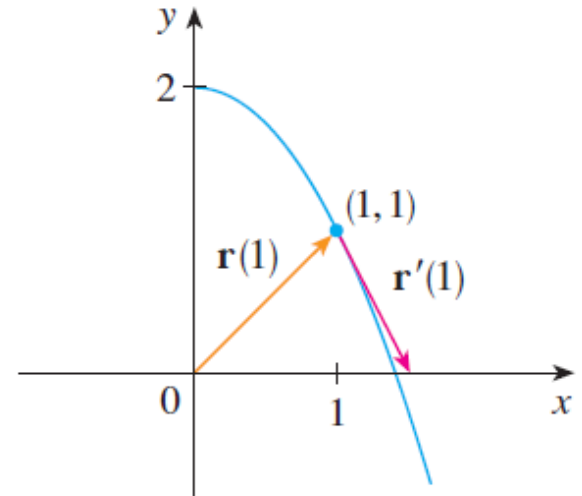
**Today:**

Arc Length followed by polar coordinates.

*Entry Task:*

$$x = t, \quad y = 2 - t^2 \quad (\text{shown below})$$

Find  $\vec{r}(1)$  and  $\vec{r}'(1)$



## Distance Traveled on a Curve

The dist. traveled from  $t = a$  to  $t = b$  is given by

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt$$
$$= \int_a^b |\vec{r}'(t)| dt$$

(Note: 2D is same without the  $z'(t)$ ).

If the curve is “traversed once” we call this **arc length**.

The distance/arc length from 0 to  $t$  is often written as

$$s(t) = \int_0^t |\vec{r}'(u)| du = \text{distance}$$

We call this the **distance/arc length function**.

Note:

$$\frac{ds}{dt} = |\vec{r}'(t)| = \text{speed}$$

*Example:*  $x = \cos(t)$ ,  $y = \sin(t)$

- Find the distance traveled by this object from  $t = 0$  to  $t = 6\pi$ .
- Find the arc length of the path over which this object is traveling.

*Example:*  $x = 3 + 2t$ ,  $y = 4 - 5t$

- Find the arc length function from 0 to  $t$ .
- Reparameterize in terms of arc length.